A Triangular Double Cantilever Beam Test for Measuring Adhesive or Cohesive Fracture Energy

G. Liu[a]; A. N. Gent[a]

[a] Department of Polymer Engineering, University of Akron, Akron, OH, U.S.A.

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A Triangular Double Cantilever Beam Test for Measuring Adhesive or Cohesive Fracture Energy

G. LIU and A. N. GENT*

Department of Polymer Engineering, University of Akron, Akron, OH 44325-0301, U.S.A.

A thin layer of adhesive bonded between two stiff elastic plates of uniform thickness and triangular in shape is recommended as a test specimen for measuring cohesive or adhesive strength. A similar test was employed many years ago by Mostovoy *et al.*[1,2] but appears to have received little attention in the intervening period. Nevertheless, it has marked advantages in comparison with current ASTM tests in simplicity of construction and use. Examples are given using silicone rubber layers bonded between steel plates.

KEY WORDS adhesion; cracking; double cantilever beam test; fracture energy; silicone rubber; triangular plate test; theory; experiment.

1 INTRODUCTION

Double Cantilever Beam (DCB) tests have been widely used to measure the strength of stiff structural adhesives[1-5] because the experiments are simple to perform and analyse. The specimens consist of two simple rectangular plates, with a layer of adhesive sandwiched between them. Linear elastic fracture mechanics may be used to compute the fracture energy, *i.e.*, the critical rate G_I of release of strain energy stored in the bent beams as the crack advances, from the applied force required to propagate the crack and some geometrical terms. However, in its simplest form the test has a serious shortcoming: the energy G_I available for fracture decreases continuously as the crack length increases. It is, therefore, necessary to measure the crack length c at any instant, as well as the applied cleavage force P or separation u of the beam ends, in order to determine the fracture energy G_I. (The critical value is denoted G_a or G_c, for adhesive or cohesive failure, respectively.)

Because of this difficulty, contoured beams have been adopted rather widely, with a cross-section chosen to give a constant relation between applied force and G_I, independent of crack length. Either the thickness or the width of the beams can

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*Corresponding author.

be varied, but it seems more usual to vary the thickness approximately in proportion to $c^{2/3}$ (when the crack length is much greater than the beam thickness) as discussed in the following section, to give a constant geometrical factor relating the applied force P to G_1.[2,4,5] However, such beams seem to be rather difficult to make in comparison with beams with constant thickness and varying width. We have, therefore, examined the feasibility of using triangular-shaped beams of uniform thickness, as shown in Figure 1, which also give a constant geometrical factor.

The width-tapered DCB test (Figure 2) was first proposed by Mostovoy et al.[1,2] Although they obviously recognized its advantages, they actually used somewhat more complicated shapes, in part to avoid possible errors arising from the assumption of a "built-in" end condition for the two beams at the crack front, and in part to allow for the application of the cleavage force at a point other than the beam tips. An example is given in Figure 2. We have used a simpler geometry, a pair of triangular steel plates, Figure 1. Metal wires brazed around the tips of the triangular plates allowed the force to be applied almost exactly at this point, without seriously affecting the elastic response of the beams. In this case a very simple expression for the fracture energy is obtained:

$$G_1 = 12P^2/EK^2D^3 \tag{1}$$

where P is the applied force to propagate the crack, E is Young's modulus of the steel plates, $K = 2\tan(\theta/2)$ where θ is the wedge angle of the plates, and D is the plate thickness. Note that the fracture force P is independent of crack length.

Measurements have been made of cohesive and adhesive strengths of test specimens prepared by bonding a flexible silicone resin between triangular steel plates. The results are compared with theoretical predictions for this geometry and with independent measurements of strength.
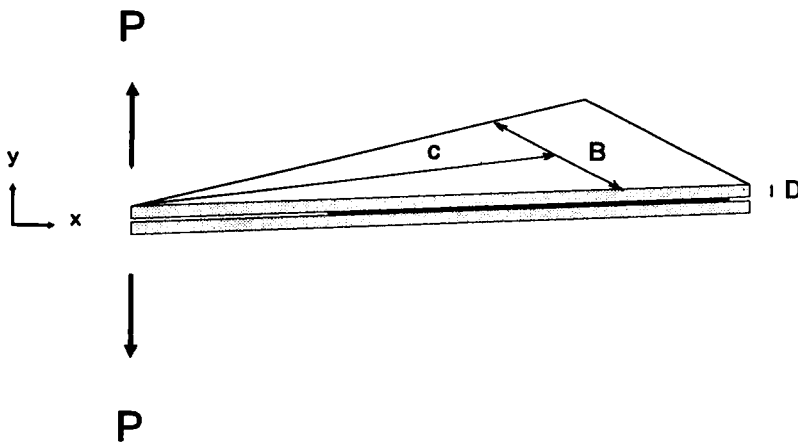

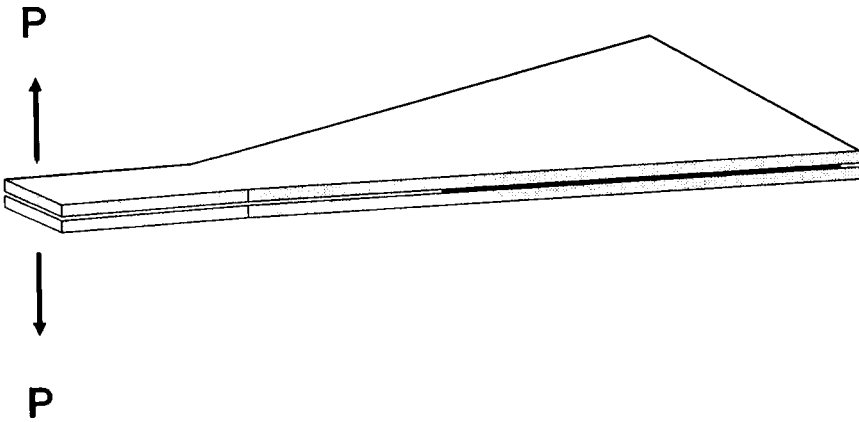FIGURE 1   A triangular double cantilever beam specimen.

FIGURE 2 A tapered double cantilever beam specimen.

2 THEORETICAL CONSIDERATIONS

For linearly-elastic systems with compliance C and crack area A, the relation between fracture energy G_I and applied force P is given by:[6]

$$G_I = \frac{1}{2}P^2(\partial C/\partial A) \tag{2}$$

For double cantilever beam specimens, with beams of constant width B, and thickness D varying with distance c from the point of application of force in the form:

$$D = K_1 c^{2/3} \tag{3}$$

where K_1 is a constant, the compliance is given by

$$C = 24c/EBK_1^3 \tag{4}$$

and the crack area $A = Bc$. Thus, the fracture energy is obtained from Equation (2) as

$$G_I = 12P^2/EB^2K_1^3 \tag{5}$$

and the fracture force P is independent of crack length c.

Similarly, for the triangular double cantilever test shown in Figure 1, the width B is proportional to the crack length c, *i.e.,* $B = Kc$ where $K = 2\tan(\theta/2)$, and the crack area $A = Kc^2/2$ (provided that the angle θ is not too large). The compliance C, given by the ratio of the separation u of the beam tips to the cleavage force P, is then obtained as:

$$C = u/P = 12c^2/EKD^3, \tag{6}$$

and the fracture energy is obtained from Equation (2) as

$$G_I = 12P^2/EK^2D^3 \tag{7}$$

as shown by Brussat *et al.*[2] Note that the fracture force is again independent of the crack length when the fracture energy is constant.

3 EXPERIMENTAL

3.1 Preparation of Steel Surfaces

Three pairs of 4140 steel plates ($\theta = 15°$, $20°$, and $30.7°$) with a length of 280 mm and thickness of 3.17 mm were roughened with sandpaper (3M Company, medium grade), washed with acetone, and dried in air. The cleaned steel plates were immersed in a solution of Primer 92023 (Dow Corning) for 1 hour and dried in air overnight to allow the solvent to evaporate. They were then heated to 80°C for 2 hours to promote the reaction between primer and steel.

3.2 Preparation of Silicone Rubber Sheets

Sylgard 184 curing agent was mixed with Sylgard 184 silicone resin (both from Dow Corning) at a concentration of 8 parts by weight per 100 parts of resin. The mixture was degassed for 30 minutes in a vacuum chamber, and then cast as a 1 mm thick sheet at room temperature for 12 hours. During this time the silicone rubber became partly cured, but it was still sticky and adhered well to other surfaces.

3.3 Preparation of Test Specimens

A thin coating of 3145 RTV adhesive (Dow Corning) was applied to two primed triangular steel plates. A previously-prepared silicone rubber sheet was then placed between the two plates, as shown in Figure 3. The assembly was held together by "C" clamps for two hours at room temperature followed by 12 hours at 60°C, and then cooled down slowly to room temperature.
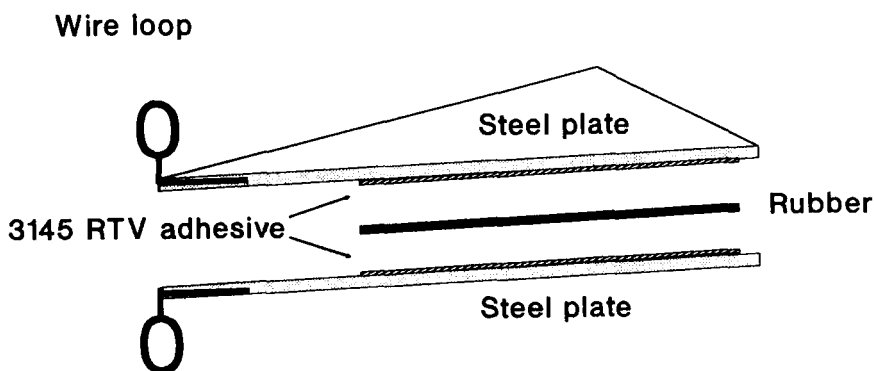

FIGURE 3   Components of a test specimen.

3.4 Test Method

Wires of 2 mm diameter were welded to the tips of the steel plates as shown in Figure 3 so that a cleavage force could be applied directly to them. Light steel chains were used to connect the wire loops to the upper and lower clamps of an Instron tensile test machine. The wide end of the specimen was supported lightly to maintain the specimen horizontal. Forces were then applied to pull the tips of the two steel plates apart at a speed of 0.5 mm/min. During the experiment the position of the support was adjusted to keep the plane of the specimen at right angles to the applied force.

4 RESULTS AND DISCUSSION

4.1 Cohesive Fracture Energy

Cohesive fracture took place approximately in the mid-plane of a well-bonded silicone rubber sheet. However, many subsidiary cracks were observed, running generally perpendicular to the main fracture plane and penetrating almost to the bonded surfaces. A representative view of part of the fracture plane is shown in Figure 4. These subsidiary cracks are tentatively attributed to fracture under the high dilatant stresses set up as the stiff steel plates are forced apart, a form of cavitation.[7] Soft incompressible solids cannot withstand dilatant stresses greater than about the value of their Young's modulus (about 2 MPa for these silicone rubber formulations) without suffering internal cracking.[8] It is noteworthy that, although the cracks ran for long distances, up to about 80 mm, following rather irregular paths, they lay generally about 10 mm apart, a distance close enough to reduce significantly the dilatant stress set up in the material lying between them. The additional work of fracture due to these subsidiary cracks was apparently rather small, probably because the additional fracture area was small in comparison with the area of the main fracture surface.

FIGURE 4   Fracture surface of silicone rubber layer, 1 mm thick. The end width of the steel plate is 100 mm.

A constant fracture force was obtained when the main fracture began to propa-
gate, Figure 5. The force was also found to be directly proportional to the value of
K (= 2tan(θ/2)), Figure 6. This is in accord with Equation (7). Values of cohesive
fracture energy G_c calculated by Equation (7) for specimens with different angles
are listed in Table I. The crack propagation rate was found to be quite constant, at
2.8 mm/min. A consistent value of cohesive fracture energy was obtained, Table I,
of about 210 J/m^2, which agrees very well with that obtained independently,
214$\pm$15 J/m^2, by tearing a pre-scored silicone rubber sheet in a trouser test at a
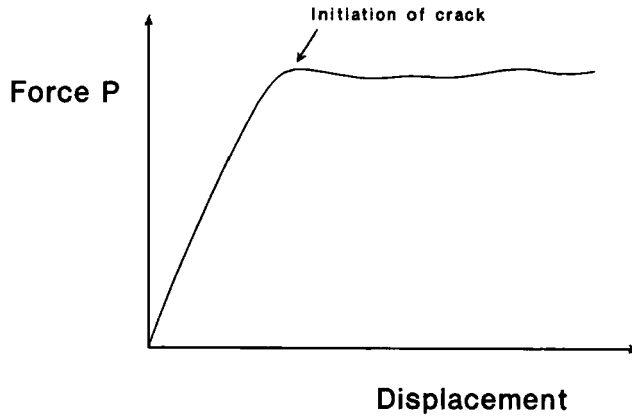rate of tear propagation of 2.5 mm/min.

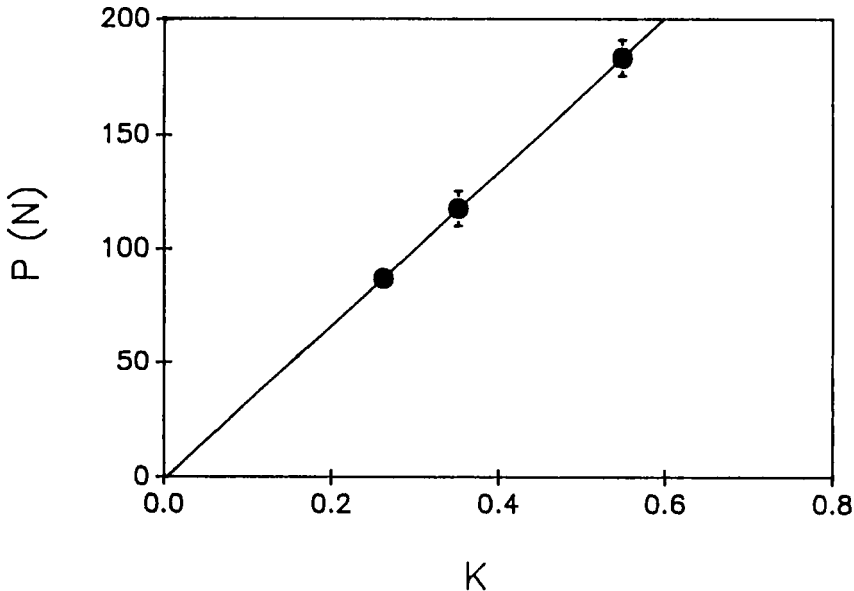FIGURE 5   Force-displacement relation for a triangular DCB test.

FIGURE 6   Fracture force plotted against plate parameter K. (K = 2tan(θ/2)).

TABLE I
Cohesive fracture energy G_c of 8 phr Sylgard 184 silicone rubber
from triangular DCB test

θ (°)	K	P (N)	G_c (J/m²)
15.0	0.263	87 ± 4	206 ± 16
20.0	0.353	118 ± 8	210 ± 21
30.7	0.549	183 ± 8	209 ± 15

4.2 Adhesive Fracture Energy

The triangular DCB test is also suitable for measuring adhesive fracture energy. When 3145 RTV adhesive was applied to only one of the primed plates, fairly good adhesion was obtained on one surface, between 3145 RTV adhesive and silicone rubber, but not on the other, between silicone rubber and steel. Adhesive fracture took place between the silicone rubber and steel surface, at a fracture force P of 75 ± 3 N at a rate of crack propagation of 7 mm/min. From Equation (7), using the relevant values of $\theta = 30.7°$ and $D = 3.17$ mm, the fracture energy G_a is obtained as only 36 ± 3 J/m².

4.3 Plate Dimensions

It is important that no plastic yielding occurs in the stiff plates under the test conditions. A minimum thickness for the plates can be estimated from simple bending theory. If a triangular plate, with length c, is bent by a constant force P, Figure 1, the maximum tension or compression stress is $\sigma_{max} = ED/2R$, where R is the radius of curvature, given by EI/M(c), and M(c) = Pc. Thus, the maximum stress is:.

$$\sigma_{max} = PcD/2I \qquad (8)$$

where $I = KcD^3/12$. Thus, from equations (7) and (8):

$$\sigma_{max} = (3G_1E/D)^{1/2} \qquad (9)$$

In our experiments the yield strength of 4140 steel is 665 MPA[9] and E is 200 GPa. G_c for silicone rubber is about 200 J/m². Thus the steel plates must have a thickness greater than about 0.5 mm to avoid yielding.

4.4 An Alternative Configuration

An alternative geometry for the tapered DCB test uses trapezoidal plates with an end width B_o, Figure 7. The cleavage force P is applied at this end of the specimen. As shown in the Appendix, the fracture energy G_1 is given by

$$G_1 = (12P^2/EK^2D^3) [1 + (9\alpha/4)]/(1 + \alpha) [1 + (3\alpha/2)]^2 \qquad (10)$$

where $\alpha = B_o/Kc$. Compared with equation (7), the factor α is introduced, leading to a dependence of the fracture force P on crack length. This dependence is small
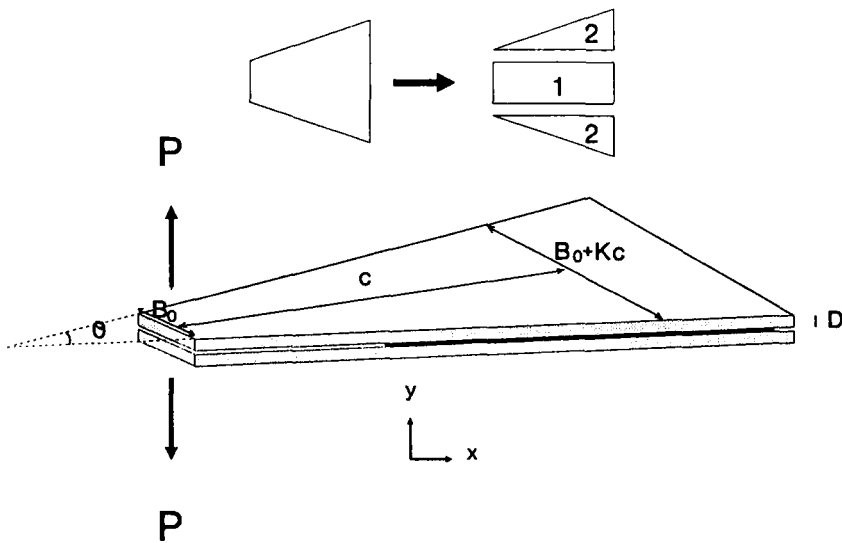
FIGURE 7   A trapezoidal double cantilever beam specimen.

TABLE II

Cohesive fracture energy G_c of 10 phr Sylgard 184 silicone rubber from trapezoidal DCB test: $B_o = 9$ mm, $D = 1.44$ mm, $\theta = 10°$ (K = 0.175)

P (N)	c (mm)	$B_o + Kc$ (mm)	α	G_c (J/m^2)
16.7 ± 0.9	168	37.5	0.316	135 ± 15
15.7 ± 0.8	176	39.3	0.297	120 ± 15
14.2 ± 0.9	201	43.5	0.261	100 ± 20

if the tip width B_o is small compared with the width Kc of plate at the crack front.

The cohesive fracture energy G_c of a similar silicone rubber (10 parts of Sylgard 184 curing agent per 100 parts of silicone resin, cured at 80°C for 6 hours) was measured in a trapezoidal DCB test at a crack speed of 0.8 mm/min. The result was 120 ± 15 J/m^2, Table II, which agrees very well with that from a trouser tearing test measured at a similar rate of tear propagation, 135 ± 5 J/m^2.

5   CONCLUSIONS

1. Triangular DCB test specimens were employed to measure the cohesive fracture energy G_c of Sylgard 184 silicone rubber. Excellent agreement was found between values of fracture energy, G_c, about 210 J/m^2, for different beam angles and in different experiments.

2. The triangular DCB test was also employed to measure adhesive fracture energy G_a between silicone rubber and a steel surface. The value was much smaller, only about 36 J/m^2.

3. Minimum thicknesses of plates to avoid plastic yielding can be calculated by means of Equation (9).

4. A slightly modified triangular DCB test, using trapezoidal plates, was also employed and gave results in good agreement with those from a trouser tear test for the cohesive fracture energy of a more highly cured silicone rubber. However, calculation is more complicated than with triangular plates because the fracture energy depends on crack length, to a greater degree the greater the initial width B_0 relative to the crack length c.

Acknowledgements

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Appendix

Trapezoidal beams are subjected to a force P, Figure 7. The compliance is calculated for a combination of a parallel-sided strip and a triangular plate with angle θ, as shown in Figure 7. Compliance for a parallel-sided strip (shown as 1 in Figure 7), is:

$$C_1 = 8c^3/EB_0D^3 \tag{11}$$

and compliance for a triangular plate (shown as 2 in Figure 7, width B = Kc), loaded at the tip, is:

$$C_2 = 12c^2/EKD^3 \tag{12}$$

Therefore, the compliance C for both pieces, is given by

$$C = 1/(C_1^{-1} + C_2^{-1}) = 12c^3/(\frac{3}{2}B_o + Kc) ED^3 \qquad (13)$$

The crack area A is $B_o c + (Kc^2/2)$, where $K = 2\tan(\theta/2)$. The fracture energy is then obtained as

$$G_I = (12P^2/EK^2D^3) [1 + (9\alpha/4)]/(1 + \alpha) [1 + (3\alpha/2)]^2 \qquad (14)$$

where $\alpha = B_o/Kc$.